Application: A very simple industrial dynamics model: Nelson & Winter (1982)

Murat Yıldızoğlu
Université de Bordeaux
http://yildizoglu.info

You are invited to develop a NetLogo model that represents a simplified version of the industrial dynamics model that is described in the following articles:

- R. R. Nelson and S. G. Winter. The Schumpetarian tradeoff revisited. The American Economic Review, 72:114–132, 1982
- Chap.12&14 in Nelson, R. and S. Winter, 1984, An Evolutionary Theory of Economic Change, Belknapp Press, Cambridge: MA

1 Elements of the model

This is a simple, one-good model of industry evolution with innovation and global imitation that implements some basic ideas about the innovation process of firms and their consequences on the dynamics of the industry. Firms behavior is conditioned by their bounded rationality (rule based decision process and inertia). Specific R&D investment is necessary for innovation. The competitive advantage of firms is based on their innovative capacity. The selection of firms is determined by their innovative capacity. Innovative behavior of firms hence determines the structure of the industry and its evolution.

1.1 Productivity, production and price

At the beginning of each period, each firm is characterized by its productivity level of the capital ($A_{i,t}$) and its capital stock ($K_{i,t}$). Capital is the only production factor and firms produce the corresponding output: $Q_{it} = A_{it}.K_{it}$. Each capital unit used by the firm costs to it c. n firms supply this good:

$$Q_t = \sum_{i=1}^{n} Q_{it} = \sum_{i=1}^{n} A_{it}.K_{it} \tag{1}$$

and they sell the totality of the production to the consumers, at a price at which the latter are ready to buy it, given their inverse demand function:

$$p_t = D\left(Q_t\right) = \frac{\mathcal{D}}{Q_t^{\eta}}. \tag{2}$$

Firms consequently obtain their gross profits:

$$\Pi_{it} = p_t Q_{it} - C\left(Q_{it}\right) = p_t Q_{it} - cK = \left(p_t - \frac{c}{A_{it}}\right) Q_{it} \tag{3}$$

which clearly express the selection force in the model: firms that increase their productivity, and expand their output, more slowly than the average suffer from a decrease of their average margin, since the decrease in the price will be stronger than the decrease of their average cost (c/A_i). If the profits are positive, a share *rdRate* of these profits is dedicated to the R&D investment, and the rest to the expansion of the physical capital.

1.2 Technical progress

Productivities (A_i) evolve as a consequence of the technical progress: innovation and imitation. Dedicated R&D investment is necessary for both processes, and the share of the R&D investment dedicated to the search for innovation is *innovShare*: $RD_{inn,t} = innovShare \times rdShare \times max\left\{0, \Pi_{it}\right\}$, while the rest of the R&D investment ($RD_{im,t}$) is dedicated to the imitation effort.

Innovation is given by a two-steps stochastic process. The probability of discovering a new innovation ($d_{in} = 1$) is determined by:

$$P\left[d_{int} = 1\right] = \underbrace{a_n}_{\text{Calibration parameter}} \underbrace{RD_{inn,t}}_{\text{R\&D investment}} \tag{4}$$

and the effective result of the innovation (the productivity discovered thanks to the innovation) is drawn from a statistical distribution (ex. Gaussian) ($F(\cdot)$):

$$\tilde{A}_{it+1} \rightsquigarrow F\left(\overline{A}_t; t, A_{it}\right)$$

This separation gives the possibility of representing different technological regimes of innovation. We will consider that each firm benefits from two types of knowledge to innovate: her productivity (her individual knowledge level), and the average productivity of the industry (collective knowledge base, $\overline{A}_t$), given the weight α of the social dimension:

$$\tilde{A}_{it+1} \rightsquigarrow \mathcal{N}\left(Z_{it}, \sigma^2\right)$$

where $Z_{it} = (1-\alpha)\, A_{it} + \alpha \overline{A}_t$. σ^2 represents the genericity of the knowledge basis (very localized innovations when σ^2 is small).

Imitation is modeled in a very simple and crude way. The probability of discovering a new imitation ($d_{im} = 1$) is determined by:

$$P\left[d_{imt} = 1\right] = a_m \underbrace{RD_{imm,t}}_{\text{Imitative R\&D}}$$

and in case of success, the firm gets the best practice of the industry (the A_t^*, maximal productivity of the period):

$$\hat{A}_{it+1} = A_{it} + d_{im} \cdot \left(A_t^* - A_{it}\right) \tag{5}$$

And, finally, the firm adopts the best currently discovered technology for the next period:

$$A_{i,t+1} = \max\left\{A_{it}, \tilde{A}_{it}, \hat{A}_{it}\right\}$$

We have also a number of firms ($nbImit$) that are pure imitators and they only invest in imitative R&D. The number of innovators is $nbInnov$ ($n = nbInnov + nbImit$).

1.3 Investment in physical capital

The investment is given by $I_t = (1 - rdShare) \times max\left\{0, \Pi_{it}\right\}$, and the new capital stock of the firm by

$$K_{i,t+1} = I_t + (1-\delta)\, K_{i,t} \tag{6}$$

where δ is the depreciation rate in this industry.

2 NetLogo program

The graphical interface should contain the following elements:

- The World will be set to the minimal size and hidden; the model is not spatial.
- Input fields for setting the parameters and initial values in the model:
  - Initial productivity of firms: $A0$;
  - Initial capital stock of firms: $K0$;
  - Unit cost of capital: c;
  - Coefficient of demand ($\mathcal{D}$): DEM;
  - Elasticity of demand (η): ETA;
  - Standard deviation of innovative draws (σ): STD;
  - Depreciation rate (δ): $DEPREC$;
  - Number of firms of each type ($nbInnov, nbImit$);
  - Number of periods: $nbPeriods$.
- A slider for selecting $\alpha \in [0,1]$ called *alpha* in the program.
- A slider for selecting $rdShare \in [0,1]$.
- A slider for selecting $innovShare \in [0,1]$.
- Two sliders for selecting the initial probabilities of innovation and imitation: $probInnov \in [0,1]$ and $probImitate \in [0,1]$.

- A button «Setup» that triggers the execution of the ***setup*** procedure (see below).
- A button «Go» that continuously call the ***go*** procedure. To stop the program, the user will just push again this button.
- A plot showing the evolution of the market price in time.
- A plot showing the evolution of productivities: average productivity ($meanProd$), as well as minimal and maximal productivities ($minProd, maxProd$).
- A plot showing the evolution of average profits for innovators and pure imitators.
- A plot showing the evolution of the market concentration in terms of output and capital stocks, using the inverse Herfindall index, giving these information in terms of equivalent number of firms equally sharing the market. For the capital stock, for example, this indicator becomes:

$$\mathcal{HK} = \frac{\left(\sum_j K_j\right)^2}{\sum_j K_j^2}, \tag{7}$$

where K_j is the capital stock of firm j. This indicator gives the equivalent number of firms that would give the same Herfindall index if each of them had the same part of capital stock:$1 \leq \mathcal{HK} \leq N$ where N is the number of active firms in the industry: $\mathcal{HK} = \mathcal{N}$ corresponding to the case where the capital is evenly shared between firms and $\mathcal{HK}$ =1 to the case where one firm owns the quasi-totality of the capital. We can compute $1 \leq \mathcal{HQ} \leq N$ using the same equation with the output of the firms.

The code of the model should contain:

- Declaration of the global variables, and the agent's variables.
- A ***setup*** procedure that initializes the model, and that creates the firms of each type. This procedure also fixes the values of a_{in} and a_{im} in order to start the simulation with the initial probabilities selected by the user, and initializes firms of both types.
- A ***go*** procedure that contains the instructions corresponding to the operations that take place in each market period, and that:
  - checks if we have already executed the program *nbPeriods* periods, and stops it if this is the case.
  - sets the individual outputs, market price and individual gross profits.
  - computes the indicators necessary for updating the plots.
  - computes the R&D investments of both types of firms and calls the reporter ***innovation*** for reading the productivity discovered thanks to the innovation, and ***imitation***, for the one learnt through this process.
  - set the maximum between these three productivities (current one, innovation, and imitation) as the productivity for the next period.
  - computes the new capital stock of the firm by calling a reporter ***investK***.
  - computes average profit for both types of firms.
  - and increases the `ticks` counter in order to transition to the next period.
- An ***innovation*** reporter that returns the productivity learnt thanks to the innovation.
- An ***imitation*** reporter that returns the current best practice if the imitation has been successful.
- An ***investK*** reporter that returns the new capital stock of the firm after investment and depreciation.
- An ***invDemand*** reporter that returns the market price given the total production of the firms.

3 Experiments

This model is quite simple. The main topic of interest is the influence of the technical progress on the performance of firms and market structure. The facility to innovate and/or imitate, as well as the social dimension of the innovation are the main factors to explore. But the parameters of the demand function, and the standard deviation of innovative draws are also interesting to study.